

# AncDE for Single-Objective Computationally Expensive Numerical Optimization With Most Appropriate Value Of Aup And Arp

Siti Khadijah, Mohd Salleh<sup>1</sup>, Diarmuid, O'Donoghue<sup>2</sup>, Abdul Samad, Shibghatullah<sup>3</sup>

<sup>1</sup>Politeknik Ungku Omar, Malaysia  
sitikhadijah@puo.edu.my

<sup>2</sup>Maynooth University, Co. Kildare, Ireland  
Diarmuid.ODonoghue@nuim.ie

<sup>3</sup>Univ. Teknologi Malaysia Melaka, Malaysia  
samad@utem.edu.my

## ABSTRACT

This paper describes an ancestral extension to the standard Differential Evolution (DE) algorithm called AncDE. We test both DE and AncDE on *CEC2015 Bound Constrained Single-Objective Computationally Expensive Numerical Optimization Problems* using *AncDE/best/1/bin* and *DE/best/1/bin*. We are using numerous value for ancestor usage probability (aup) and ancestor replacement probability (arp) to define which value would produce the best result for each functions. Generally, the result show that AncDE outperformed standard DE. Meanwhile, the values that used in *aup* and *arp* are significant with types of function complexity as well as size of dimension.

**Keywords:** Different vector, ancestor archieve, ancestor usage probability, ancestor replacement probability.

## 1. Introduction

Differential Evolution (DE) has become extremely popular because of its efficiency and simple implementation and has been shown to be one of the most reliable algorithms in dealing with optimization problems [1]. In standard DE with *best/1/bin*, a vector called the base are randomly selected from initial population  $G = \{x_1, x_2, \dots, x/G\}$ . Then the other two distinct vectors are selected from the population  $G$  and calculate the difference vector between them. This different vector then multiplies with  $F$ , a mutation factor in  $[0, 2]$  that controls the extension of differential variation, then added with the base vector to produce *donor vector* [2]. Crossover phase produces a trial vector using binomial crossover:

$$u_{j,i,G} = \begin{cases} u_{j,i,G}, & \text{if } (rand_j[0,1] \leq CR) \text{ or } (j = j_{rand}) \\ x_{j,i,G}, & \text{otherwise} \end{cases} \quad (1)$$

CR is crossover constant in range  $[0,1]$  while  $j_{rand}$  in randomly chosen integer in range  $[1,D]$  to ensure the trial vector  $U_{i,G}$  is differ from related target vector  $X_{i,G}$

at least in one dimension. Last stage is selection phase, and all stage repeated until it reaches the stopping criteria [3]:

$$X_{i,G+1} = \begin{cases} U_{i,G}, & \text{if } f(U_{i,G}) \leq f(X_{i,G}) \\ x_{i,G}, & \text{otherwise} \end{cases} \quad (2)$$

One of the benefits of difference vectors is that the magnitude of moves decreases as the population converges to an optimum, allowing a more fine-grained search to occur [4]. Therefore by applying new regions of the solution space that associated with random mutations of difference vector, it will allow large jumps in the solution space to uncover a new path to the global optimum.

### 1.1 Ancestral Differential Evolution –AncDE

AncDE has proposes the following extension to the standard Differential Evolution: first, a second shadow population repository of discarded vectors of current population when new trial vectors enter the population, which are updated stochastically. Second, use this population of ancestor vectors when generating donor vectors. This property allows the use of current and ancestral population in difference vectors to broaden the search thus offers facilitating the exploration of the solution space. There are two parameters introduced in AncDE to control the ancestor influence upon the current population: ancestor usage probability (aup) and ancestor replacement probability (arp). Ancestor usage probability (aup) moderates the frequency that an ancestor is used to calculate the difference vector.

The ancestor replacement probability (arp) controls the relative age of the ancestral population. The version of AncDE in this paper is based on DE/best/1/bin but the techniques can be easily applied to most variants of DE. At mutation stage, AncDE is using AncDE/best/1/bin will select a base vector  $\vec{X}_i$  from the current population, and using ancestor usage probability (aup) to select ancestral vector  $A(\vec{X}_i)$ . AncDE has proposed a new ancestral difference vector which differ from DE to calculate the difference vector when random value k is smaller than aup:

$$\vec{V}_{i,j} = \vec{X}_i + F(A(\vec{X}_i) - \vec{X}_i) \quad (3)$$

AncDE will retain normal difference vector when the random value k is bigger than aup. In crossover stage, for binomial crossover we use the following formula with  $p = Cr$  and each j is an element of a vector  $\vec{X}_i$ :

$$\vec{U}_{i,j} = \begin{cases} \vec{V}_{i,j} & \text{if } rand(0,1) \leq Cr \text{ or } \leq i = j \\ \vec{X}_{i,j} & \text{otherwise} \end{cases} \quad (4)$$

AncDE uses the same selection and replacement process as standard DE. However, AncDE had added additional step; if the new trial vector is better than the current base vector and the random value j is lower than ancestor replacement probability (aup) then the ancestral archive will be updated, but if random value j is

bigger than ancestor replacement probability (*aup*) then the ancestral archive will remain the same. All stage repeated until it reaches the stopping criteria [2].

## 1.2 AncDE Parameters - *Aup* and *Arp*

Ancestor usage probability (*aup*) and ancestor replacement probability (*arp*) are additional parameters that influence the result of AncDE. Ancestor usage probability (*aup*) is to controls the frequency of ancestral differences vector and the value is between [0,1]. If the *aup* = 0, the ancestral difference vector will not be applied and reduce the possibility of having ancrestral template in the next population. However, *aup* value that closes to 1 will increase the impact or archive vector into the current population. Meanwhile, for ancestor replacement probability (*arp*) that controls the age of relative ancestor archive. The *arp* value that too low will cause the ancestor archive unrelated with the current population, however the higher value of *arp* will make the ancestor archive too similar with the current population. The aim of this paper is to define the caractheristic of *aup* and *arp* based on types of function.

## 2. RESULT

We had run AncDE and DE on *CEC2015 Bound Constrained Single-Objective Computationally Expensive Numerical Optimization Problems* with *DE/best/1/bin* and *AncDE/best/1/bin*. For 10D, both DE and AncDE were used NP = 12, F = 0.6 and CR = 0.75. Meanwhile, for 30D we used NP = 25, F = 0.7 and CR = 0.5, with range  $\pm 0.75$  for both 10D and 30D. We used the same setting is because we would like to focus on the impact of various value of *aup* and *arp* for each function in every dimension. We had used 10 values between 0.05 until 0.5 for both *aup* and *arp*.

Table 1 and Table 2 shows the details of result produced by AncDE and DE. Numbers in boldface in both tables displayed better result; AncDE outperformed with 99% of DE for 10D while AncDE got 100% victory over DE for 30D. Since we used the same setting of F and CR on both methods, therefore we believe this is a fair comparison.

On the other hand, the result from various values of *aup* and *arp* were significant with the function complexity and size of dimension. As we run each *aup* versus *arp* for every function for 10D as shown in Figure 1, literally the highest *aup* is 0.05 and 0.3 for *arp*. Meanwhile, Figure 2 shows the result of *aup* and *arp* usage in 30D for each function, where the highest *aup* usage is 0.35 and 0.1 for *arp*. The best combination of *aup* and *arp* for 10D is 0.25 | 0.3 and 0.4 | 0.35 where both produced better result with highest number of functions. For 30D, the best combination is 0.5 for *aup* and 0.1 for *arp*.

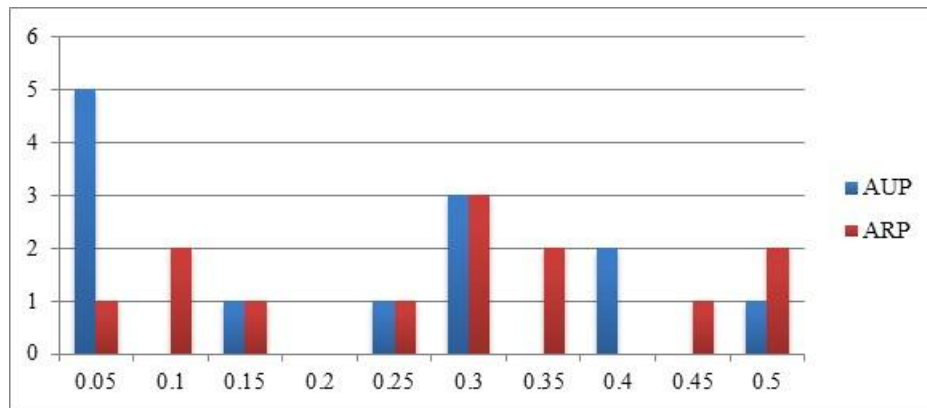


Figure 1: Numbers *aup* and *arp* used for each functions in 10D

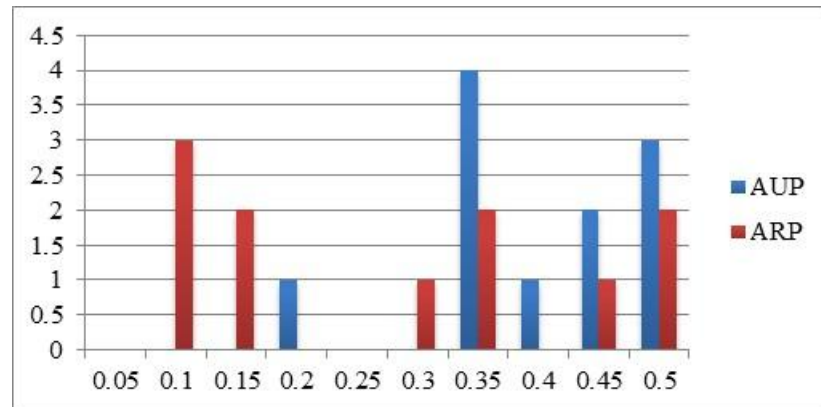


Figure 2: Numbers *aup* and *arp* used for each functions in 30D.

Table 1: Result for both DE and AncDE for 10D.

Standard DE			AncDE		
Func	Median	Mean	Median	Mean	St.Dev.
1	7.40E+07	1.73E+09	<b>5.06E+06</b>	<b>3.33E+06</b>	5.65E+06
2	4.31E+04	1.58E+07	<b>1.65E+04</b>	<b>1.58E+04</b>	4.85E+03
3	3.08E+02	3.08E+02	<b>3.03E+02</b>	<b>3.03E+02</b>	3.88E-01
4	<b>2.28E+03</b>	<b>2.36E+03</b>	5.06E+06	3.33E+06	5.65E+06
5	5.04E+02	5.04E+02	<b>5.02E+02</b>	<b>5.02E+02</b>	2.33E-01
6	6.02E+02	6.03E+02	<b>6.00E+02</b>	<b>6.00E+02</b>	7.41E-02
7	7.04E+02	7.15E+02	<b>7.00E+02</b>	<b>7.00E+02</b>	8.94E-02
8	8.11E+02	2.92E+04	<b>8.04E+02</b>	<b>8.04E+02</b>	9.59E-01
9	9.04E+02	9.04E+02	<b>9.04E+02</b>	<b>9.04E+02</b>	2.04E-01
10	2.22E+05	3.51E+06	<b>3.58E+04</b>	<b>3.16E+04</b>	1.62E+04
11	1.11E+03	1.13E+03	<b>1.10E+03</b>	<b>1.10E+03</b>	6.18E-01
12	1.58E+03	1.65E+03	<b>1.32E+03</b>	<b>1.31E+03</b>	4.90E+01
13	1.62E+03	1.71E+03	<b>1.50E+03</b>	<b>1.61E+03</b>	1.58E+02
14	1.61E+03	1.61E+03	<b>1.59E+03</b>	<b>1.60E+03</b>	3.51E+00
15	1.94E+03	1.96E+03	<b>1.52E+03</b>	<b>1.51E+03</b>	2.49E+01

Range of best *aup* value for 10D is between 0.25 and 0.4, while in *aup* is between 0.3 and 0.35. The best range of *aup* for 30D is between 0.35 and 0.5, while *arp* is between 0.1 and 0.5. Based on the result in 10D, less complexity functions tends to use smaller *arp* value to produce better result with scattered *aup*, which differ with more complex functions that using *aup* and *arp* in between 0.25 and 0.35. However, in 30D, all functions were relying *aup* in range 0.35 until 0.5 and more complex functions were located in range 0.1 and 0.15 for *arp*.

Table 2: Result for both DE and AncDE for 30D.

Standard DE			AncDE		
Func	Median	Mean	Median	Mean	St.Dev.
1	2.11E+10	2.91E+10	<b>3.44E+08</b>	<b>3.72E+08</b>	1.40E+08
2	1.70E+05	4.45E+05	<b>9.22E+04</b>	<b>8.92E+04</b>	1.57E+04
3	3.39E+02	3.40E+02	<b>3.30E+02</b>	<b>3.30E+02</b>	2.59E+00

4	7.33E+03	7.90E+03	<b>6.32E+03</b>	<b>6.35E+03</b>	2.45E+02
5	5.05E+02	5.05E+02	<b>5.03E+02</b>	<b>5.03E+02</b>	3.71E-01
6	6.03E+02	6.03E+02	<b>6.01E+02</b>	<b>6.01E+02</b>	5.28E-02
7	7.34E+02	7.60E+02	<b>7.00E+02</b>	<b>7.00E+02</b>	5.18E-02
8	5.36E+05	7.88E+06	<b>1.03E+03</b>	<b>1.06E+03</b>	1.42E+02
9	9.14E+02	9.14E+02	<b>9.13E+02</b>	<b>9.13E+02</b>	1.43E-01
10	1.22E+08	1.26E+08	<b>1.57E+07</b>	<b>1.72E+07</b>	6.17E+06
11	1.33E+03	1.38E+03	<b>1.15E+03</b>	<b>1.15E+03</b>	1.23E+01
12	3.34E+03	1.11E+04	<b>2.46E+03</b>	<b>2.41E+03</b>	1.72E+02
13	1.77E+03	1.95E+03	<b>1.67E+03</b>	<b>1.67E+03</b>	1.15E+01
14	1.83E+03	1.85E+03	<b>1.68E+03</b>	<b>1.68E+03</b>	1.58E+01
15	2.87E+03	2.92E+03	<b>2.56E+03</b>	<b>2.53E+03</b>	9.93E+01

### 3. CONCLUSIONS

This paper presented an ancestral extension of standard DE called AncDE with generated a solution archive that updated stochastically from the current population. AncDE used a additional ancestor difference vector mechanism to extent better solution possibility. AncDE also use ancestor usage and replacement probability as parameter controller of ancestor vector age and usage. DE and AncDE has tested on *CEC2015 Bound Constrained Single-Objective Computationally Expensive Numerical Optimization Problems*, and the result shows that AncDE outperformed DE on most of the functions. Besides, we also can conclude the *aup* and *arp* value are significant with functions complexity as well as the dimension's size.

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